Discussion of "Simplified Two-Parameter Gamma Distribution for Derivation of Synthetic Unit Hydrograph" by P. K. Bhunya, S. K. Mishra, and Ronny Berndtsson

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Replacing Two Empirical Expressions with a Single Expression

The concept of a cascade of n linear reservoirs gives rise to an interesting unit hydrograph (UH) given by Eq. (1) in the original paper, which is repeated here for convenience:

$$q = \frac{1}{K\Gamma(n)}\left(\frac{t}{K}\right)^{n-1} e^{-t/K} \tag{1}$$

The authors are perfectly right in trying to simplify the relationship between the conceptual parameters n and K of this UH and its two major graphic characteristics, that is, its peak value, noted q_p, and the time of peak occurrence, t_p. The relationship is very simple for t_p (Eq. 2) of the original paper but far more complicated for q_p (Eq. 3) of the original paper;

$$\beta = q_p t_p = \frac{(n-1)^{(n-1)} e^{-(n-1)}}{\Gamma(n-1)}$$

To clarify the relationship between β and n and to make it possible to easily compute n from β, Eq. (3) of the original paper must undergo some simplification.

To do so, Stirling's formula (Abramowitz and Stegun 1972) for the gamma function can prove useful:

$$\Gamma(x) \approx e^{-x}x^x\sqrt{\frac{2\pi}{x}}\left(1 + \frac{1}{12x} + \frac{1}{288x^2} - \cdots\right) \tag{2}$$

The term inside the parentheses begins as

$$\sqrt{1 + \frac{1}{6x}}$$

Then, a simplified form of Stirling's formula is

$$\Gamma(x) \approx e^{-x}x^{x-1}\sqrt{2\pi\left(x + \frac{1}{6}\right)}$$

Using this expression in Eq. (3) of the original paper gives

$$\beta \cong \frac{n-1}{\sqrt{2\pi(n-1) + \frac{1}{6}}}$$

It must be noted that basic algebra concerning Eq. (3) reveals that β is equivalent to $n-1$ when this quantity is close to zero. Therefore, it is advisable to accept being less accurate for large values of n and replace $1/6$ with 1 in the above approximation, giving β to conform with its behavior when n is around 1. This change yields the following approximation for β, which is largely sufficient for hydrological applications:

$$\beta \cong \frac{n-1}{\sqrt{2\pi(n-1)+1}} \tag{3}$$

This expression is very simple and is more suggestive of the overall behavior than the exact but cumbersome expression given by Eq. (3) of the original paper. Its major merit is that it is invertible to give n in function of β:

$$n \cong 1 + \pi\beta^2 + \beta\sqrt{1 + (\pi\beta)^2} \tag{4}$$

I suggest that this single expression could replace Eqs. (8) and (9) proposed by the authors. Another benefit arises when expressing n as a function of λ. Since $\lambda = \beta/(n-1)$, Eq. (2) in this discussion directly yields the following expression:

$$n \cong 1 + \frac{1}{2\pi}\left(\frac{1}{\lambda^2} - 1\right) \tag{5}$$

This equation could replace Eq. (10) and (11) in the original paper as well, thereby avoiding the introduction of new and conflicting approximations; Replacing λ with $\beta/(n-1)$ in Eq. (10) in the original paper results in a new expression in conflict with Eq. (8).

Variable or Constant Parameters?

The assertion by the authors that n and λ depend on storm characteristics could be highly confusing. In essence, all parameters of the UH are dependent only on the physical characteristics of the studied catchment and should be constant for every storm. There is no reason in their development to infer any relationship with storm characteristics. The dependence on the observed hydrograph intervenes only when calibrating the UH parameters on observed rainfall-runoff data, as for any model. However, the most reliable estimates are obtained when pooling several storms together in the calibration process.

Unit Hydrograph Ordinates Units

Units used on Figs. 2 to 5 of the original paper on the y-axis, i.e., Q in cubic feet per second (cfs) are regrettably confusing. If $I(t)$ denotes the intensity of the effective rainfall as a function of time t, the discharge computed using the UH $q(t)$ is given by

$$Q(t) = \int_0^t I(u)q(t-u)du$$

When I is expressed in mm/h, q in h^{-1}, and t in h, then Q is also expressed in mm/h. Let us denote this number as Q_1. Considering the area A (km^2) of the catchment, Q can be expressed in $m^3\,s^{-1}$ to become equal to $Q_2 = Q_1 \cdot A/3.6$. It could be tempting to include this dimensions-related coefficient into the expression of the UH to give a more dischargelike expression of the UH:

$$q_2(t) = \frac{A}{3.6} q(t)$$

However, one has to realize that q_2 is not in $m^3 s^{-1}$ but in $m^3 s^{-1} mm^{-1}$, i.e., in $10^3 m^2 s^{-1}$, provided the other units remain the same ($mm h^{-1}$ for I and h for t). Accordingly, the cfs cannot be a suitable unit for q_2.

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The authors should be appreciated for using the method proposed by Singh (2000) for transmuting popular synthetic unit hydrographs (SUHs) [i.e., those of Snyder (1938), Gray (1961), and SCS (1972)] into a two-parameter gamma distribution, which avoids the error associated with a manual fitting over a few points and adjusting for a unit runoff volume. Derivation of an SUH more conveniently and accurately than the popular SUHs is available [see Singh (2000)]. Before taking up discussion on the approximations developed by the authors and on their findings, it would be appropriate to discuss the available method. Singh (2000) also showed that the popular SUHs are transmutable to a two-parameter gamma distribution and gave a unified conceptual interpretation of the coefficients appearing in the equations of the popular SUHs. He used two independent parameters β and t_p for transmuting Snyder's and SCS's synthetic unit hydrographs into a gamma-distribution:

$$n = \frac{7}{6} + 2\pi\beta^2 \tag{1}$$

$$\beta = q_p t_p \tag{2}$$

$$\beta = C_p\left(1 + \frac{t_r}{2t_l}\right) \tag{3}$$

$$\beta = \frac{D_f}{645} \tag{4}$$

The authors seem to have used Eqs. (2)–(4) in the Application section but did not to mention it. In Application, it is not clear how the value of t_p was obtained from Snyder's C_p in Case A. Similarly, it is not clear how the value of t_p was obtained in Case C. Similar questions also arise in Case D.

The version of the gamma distribution proposed by the authors is incomplete because it discusses only one parameter (say, n). Although they proposed its relation with two other related parameters (β and λ), a two-parameter gamma distribution needs to have two independent parameters.

Relation between *n*, β, and λ

The authors proposed two equations that use a numerical optimization for relating n and β [authors' Eqs. (8) and (9)], applicable for different ranges of β; Eq. (8) is applicable for $0.01 < \beta < 0.35$, and Eq. (9) is applicable for $\beta \geq 0.35$. The approximate equation already proposed by the discusser [Eq. (1) of this discussion; see also Singh (2000)] is derived analytically, and which converges asymptotically to the true equation. Eqs. (8) and (9) in the original paper are similar to Eq. (1) of this discussion and are not easily invertible, whereas Eq. (1) is easily invertible and can also be used to obtain the value of β for a known value of n (such a case may arise when n and q_p or t_p are known). One can use the following single equation proposed by Singh (1998) in place of the authors' two equations [Eqs. (8) and (9)].

$$n = 1.09 + 0.164\beta + 6.19\beta^2 \tag{5}$$

Singh (2000) observed that β generally varies between 0.35 and 1.25; therefore, Eq. (8) does not have a much practical utility. This result is also observed in the examples considered by the authors. Although the authors' Eq. (9) gives less error for $0.35 \leq \beta \leq 0.7$ than Eq. (1) of this discussion it is trivial (or very small) and does not have any improvement on the practical application in obtaining a gamma SUH. Table 1 gives the comparison of errors. The authors have not demonstrated the improvement in computed SUHs over that obtained using the Singh (2000) method for the examples considered in the Application section. In fact, the method of Singh (2000) yields equally good results.

The parameter λ may not be required for obtaining a gamma SUH from the parameters of the popular SUHs. In the example (Case 3), n can be determined from the empirical equation proposed by Nash (1960), who proposed empirical equations for both K and n. However, the following analytical approximation is developed for obtaining the value of n for a known value of λ:

$$n = \frac{1}{2\pi\lambda^2} + \frac{5}{6} \tag{6}$$

Eq. (6) is an analytically derived approximation and can be inverted easily, similar to Eq. (1). Inversion of Eq. (6) or (1) is required if a gamma SUH is obtained for given values of n and q_p or t_p. Eq. (6) gives adequately accurate results for the practical range of λ (≤ 0.25), which corresponds to $\beta \geq 0.35$. From Eqs. (1) and (6), one obtains

$$\lambda = \sqrt{\frac{3}{2\pi(1 + 6\pi\beta)}}; \qquad \beta \geq 0.35 \quad \text{or} \quad \lambda \leq 0.25 \tag{7}$$

The errors in the computed values of n using Eq. (1) in this discussion and Eq. (10) in the original paper are the same as given in Table 1, with β being replaced by corresponding λ using Eq. (7).

It is painful to observe that the authors have drawn a negative conclusion that n and β both depend on the storage characteristics of the basin. If n is expressible in terms of β and λ, λ depends only on the catchment characteristics, as both n and β do [see Singh (2000, p. 382, paragraph below Eq. (17)]. Although the

Table 1. Comparison of Errors in Computed Values of n

	Percent error in n	
β	Eq. (9) (from original paper)	Eq. (1) (from this discussion)
0.35	0.6	1.0
0.50	0.1	0.3
0.60	0.1	0.2
0.70	0.1	0.1
0.80	0.0	0.1
0.90	0.0	0.0
1.00	0.0	0.0
1.10	0.0	0.0
1.20	0.0	0.0
1.25	0.0	0.0

parameter K or t_p contains the storage property of the catchment, the parameter λ ($=q_pK$) is independent of K or the storage property of the catchment. This can also be shown mathematically by substituting t_p from the authors' Eq. (2) into Eq. (3). It is further stated that the parameters n and K are independent that the parameters, β and t_p are independent. Again, the authors' inference "for a given q_p, the t_p increases with a decrease in K" is inconsistent. Eqs. (2) and (3) in the original paper show that, for a given q_p, if t_p is considered to vary, n would vary; but for a gamma SUH, n is a parameter and needs to be constant. In fact, for a gamma SUH, if q_p is constant, t_p will also be a constant and these two are a set of parameters of a two-parameter gamma distribution. In Case A, the authors have considered this set of parameters. Therefore, the observation of Valdes et al. (1979) and Singh (2000) that the n or β depends only on the physical characteristics of the catchment and not on the storage characteristics is justified.

Although readers can well correct the typographical errors they need to be corrected. For example; "Singh (1988, 2000)" twice appearing on page 227 may have been intended to be "Singh (2000);" and on the same page, in the paragraph below Eq. (11), "β values less than 0.01 are" should be "β values less than 0.35 are."

Conclusions

The following conclusions are drawn:

1. A gamma SUH cannot be obtained by using only one parameter, i.e., n, β, or λ (these are effectively one parameter because n can be expressed in terms of β or λ).
2. The parameters n and β depend only on physical characteristics of the catchment and not on the storage characteristics.
3. Using the authors' equations for obtaining n does not offer an improvement in the computed SUH over the use of the equation proposed by Singh (2000) for the practical and realistic range of the parameters. The added advantage of Eqs. (1) and (6) over those proposed by the authors is that these are invertible and have a wider application.

Notation

The following symbols are used in this discussion:

C_p = peak rate coefficient in Snyder's (1938) SUH (dimensionless);
D_f = peak rate factor in SCS's (1972) SUH (dimensionless);
K = storage coefficient of linear reservoirs in Nash (1960) model (T);
n = number of linear reservoirs in Nash (1960) model (dimensionless);
q_p = peak of SUH for unit effective rainfall (T^{-1});
t_l = catchment lag measured from center of effective rainfall to peak of SUH (T);
t_p = time to SUH peak (T);
t_r = duration of effective rainfall (T);
β = shape parameter of SUH (dimensionless); and
λ = another shape parameter of SUH (dimensionless).

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Closure to "Simplified Two-Parameter Gamma Distribution for Derivation of Synthetic Unit Hydrograph" by P. K. Bhunya, S. K. Mishra, and Ronny Berndtsson

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The writers appreciate the discussers for helping generate audience interest in the paper and providing innovative and useful suggestions. They also agree with the point that the Stirling's formula surrogates $\Gamma(x)$ [Eqs. (8) and (9)] lead to the (approximate) form of the two-parameter gamma distribution. Furthermore, this does not lead to an exact solution for n with known β. The accuracy of Eqs. (4) and (5) in Michel's discussion vis-à-vis Eqs. (8) and (9) can be better explained with the aid of Table 1.

Apparently, Eq. (4) of Michel's discussion yields a larger error than Eqs. (8) and (9), except for lower values of β, which are seldom exhibited by real flood events. The equation given by the discusser (Eq. 5) was actually simplified to Eq. (1) below for greater accuracy, as follows:

Table 1. Error in Computed n Using Eq. (4) of Michel's Discussion

	Nash parameter (n)			Error (%)=100×(actual–computed)/computed	
β	Actual	Eq. (8) (original paper)	Eq. (4) of Michel's discussion	Eq. (9) (original paper)	Eq. (5) of Michel's discussion
0.0421	1.05	1.06195	1.047983	–1.138093	0.192114
0.2420	1.50	1.501197	1.487887	–0.079832	0.807520
0.3679	2.00	2.000082	1.987401	–0.004103	0.629960
0.4625	2.50	2.504786	2.488031	–0.191422	0.478772
0.5265	2.90	2.902878	2.888539	–0.099229	0.395221
0.5413	3.0	3.002550	2.988654	–0.084992	0.378192
0.6102	3.5	3.501417	3.489160	–0.040495	0.309705
0.6721	4.0	4.000783	3.989562	–0.019565	0.260955

$$\beta = \frac{n-1}{\sqrt{2\pi(n-1)+1/6}} \tag{1}$$

or

$$n \cong 1 + \pi\beta^2 + \beta\sqrt{1+(\pi\beta)^2} \cong 7/6 + 2\pi\beta^2$$

Singh's viewpoint that Eq. (1) in his discussion is more flexible in inversion is not entirely true, and it can be shown by rewriting Eqs. (8) and (9) of the original paper as a function of n to compute β, as follows:

$$\beta = [(n-1.04)/5.53]^{1/1.75}; \quad 0.01 < \beta < 0.35 \tag{2}$$

$$\beta = [(n-1.157)/6.29]^{1/1.998}; \quad \beta > 0.35 \tag{3}$$

The error in computing β is given in Table 2. It is clear from Table 2 that the use of Eq. (1) in Singh's discussion yields significantly higher error than Eqs. (2) and (3) of this closure. In addition, the computation of β is not possible when n nears 1. Thus, Eqs. (2) and (3) are more flexible, and the resulting error is comparatively less. Nevertheless, all the formulas derived so far by various authors exhibit approximate relations rather than exact analytical solutions of Eq. (1) of the original paper. The error in computing n from λ by using Eqs. (4) of Michel's discussion and (7) of Singh's discussion is more compared with Eqs. (10) and (11) of the original paper, as shown in Table 3.

Table 2. Error in computation of β.

	β			Error (%)=100×(actual–computed)/computed	
N	Actual	Eq. (1) of Singh's discussion	Eqs. (2) and (3) of this closure	Eq. (1) of Singh's discussion	Eqs. (2) and (3) of this closure
1.05	0.04	CNC[a]	0.03	—	—
1.50	0.24	0.23	0.24	4.81	0.20
2.00	0.37	0.36	0.37	1.00	0.59
2.50	0.46	0.46	0.46	0.41	0.18
3.00	0.54	0.54	0.54	0.22	0.07
4.00	0.67	0.67	0.67	0.09	0.01
5.00	0.78	0.78	0.78	0.05	0.00

[a]CNC=Cannot be computed.

Table 3. Error in Computed λ Using Eq. (4) of Michel's Discussion.

	Nash parameter (λ)			Error (%)=100×(actual–computed)/computed	
λ	Actual	Original paper	Eq. (4) of Michel's discussion	Original paper	Eq. (4) of Michel's discussion
0.501994	1.3	1.381105	1.472417	–6.238860	–13.2629
0.466714	1.5	1.466809	1.571511	2.212737	–4.767380
0.366091	2.0	2.009828	2.028371	–0.491390	–1.418560
0.308175	2.50	2.522848	2.516656	–0.913910	–0.666250
0.270837	3.0	3.005101	3.010563	–0.170030	–0.352120
0.244354	3.50	3.502728	3.506369	–0.077950	–0.181980
0.224346	4.00	4.001325	4.002993	–0.033130	–0.074820
0.195682	5.00	4.999876	4.997235	0.002473	0.055306

Authors also assert that λ may not be required in obtaining an SUH as such.

Relation among *n*, λ, and β

It will be appreciated that β is a product of q_p and t_p, therefore, it will not be the same for different (real) storms of different q_p and t_p values. However, there can be a number of q_p–t_p sets for which β is the same. The question of whether β varies with storm characteristics needs more research for a rational explanation. However, with reference to Eq. (12) of the original paper, for a constant value of q_p for a storm, t_p can be related to λ ($=q_pK$) or K by the following relation:

$$\lambda = \frac{0.636}{1+4.13(\beta)^{1.52}} + 0.029; \quad \beta \geqslant 0.54, \lambda \leqslant 0.27 \tag{4}$$

Therefore, since K is influenced by the incipient moisture conditions and in situ basin storage, β will also be influenced by these factors.

Unit of *q*

In the original paper, the discharge or runoff (q) is taken as the equivalent depth of instantaneous flow per unit time step, and its unit is mm/hr/mm or inch/hr/inch. Such a definition helps make the term β nondimensional.

Finally, the authors gratefully acknowledge that typographical errors were inadvertently introduced and therefore suggest that the readers incorporate the corrections pointed out by the discusser.

Discussion of "Simple Snowdrift Model for Distributed Hydrological Modeling" by M. Todd Walter, Donald K. McCool, Larry G. King, Myron Molnau, and Gaylon S. Campbell

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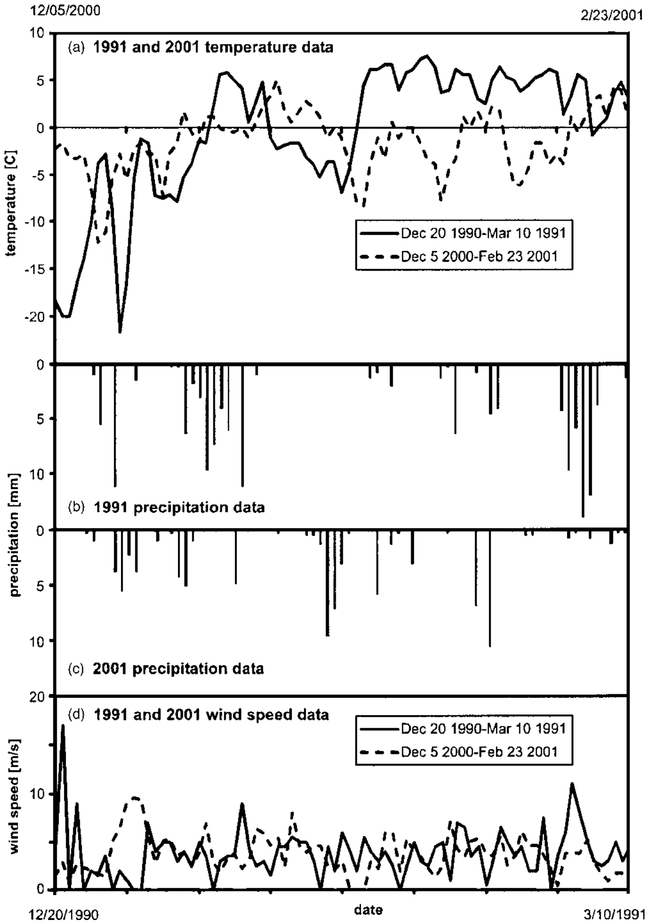

Fig. 1. Summary of the Walter et al. (2004) study period (1990–1991) to the comparison study period (2000–2001): (a) temperature; (b) and (c) precipitation; and (d) wind-speed data when humidity data were available

Although there has been extensive research on snow drifting, specifically its importance in water resources and the understanding of the process, it has received limited use and implementation in hydrological modeling. Many wintertime distributed models have traditionally limited the scope to snow accumulation and snowmelt, and the authors give several examples (DVHSM, SHE, SWAT). Wind movement of snow into, out of, and within a watershed can substantially alter the distribution of snow, and decrease the net mass of snow through sublimation losses during transport. In open terrain, the movement and sublimation of snow has been shown to be significant (Pomeroy and Li 2000). Winstral et al. illustrated that changes in topography in the upwind direction can be used to model the distribution of snow.

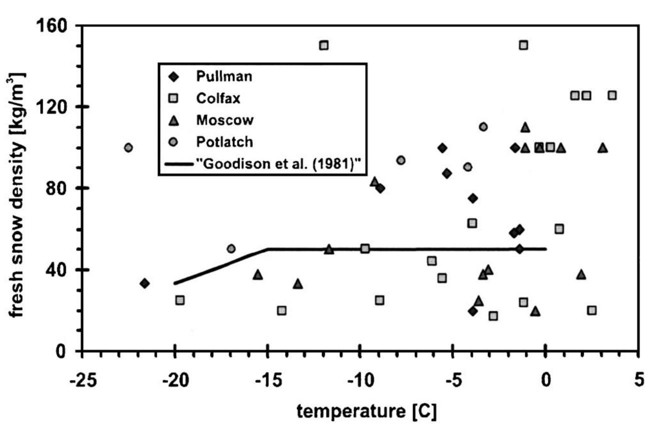

Fig. 2. Fresh-snow density estimates for the Walter et al. (2004) study sites and surrounding locations

The authors present a simple snowdrift model that is meant to be easier to use than more complex models, such as PBSM (Pomeroy et al. 1993), Piektuk (Déry et al. 1998), and SNOWTRANS-3D (Liston and Sturm 1998). The main purpose is to be able to incorporate the snow-blowing process into a distributed hydrological model with an emphasis on "*simplicity… to avoid overly complex parameterization*" (Walter et al. 2004).

The author's snowdrift model can move any fresh snow that is on the ground (drift of falling snow is not considered) if the wind speed exceeds a threshold that is based on the snow's shear velocity, which is a function of the square root of the fresh snow density. The fresh snow density is computed from Goodison et al. (1981); for air temperatures (T_a) warmer than $-15°C$ it is set at 50 kg/m^3 and for colder temperatures it is computed as $50+3.4(T_a+15)$. Snow leaves the surface at disruptions in the wind profile, in particular on the lee side of a hill or end of vegetation. The model uses a two-layer snowpack where new snow overlays old snow; upon densification greater than 150 kg/m^3, new snow is incorporated into the old snow layer and is not available for movement. When drifting occurs, the model assumes sublimation from the snowpack surface, which can be up to twice the snow flux moved by drifting. The model was applied to an isolated hill 10 km northeast of Pullman, Wash., for the period from December 20, 1990, through March 10, 1991, and is compared to a series of eight days of snowpack measurements. The forcing meteorological data for the study was taken from a station 3 km Northwest of Pullman. The model accumulates snow at air temperatures colder than 0°C from precipitation data that has not been corrected for snowfall-measurement inaccuracies, such as wind undercatch.

The results presented in Fig. 3 of Walter et al. (2004), illustrate that the model tends to underestimate snow water equivalent (SWE) except in areas of drift accumulation. The snowdrift model matched the observed spatial and temporal trends across the study hill slope with an r^2 of 0.95, compared with a no-drift simulation correlation of 0.33. The differences between measured and observed snowdrift results were the same magnitude as modeled snowmelt errors. The results of the sensitivity analysis found the most important factor to be fresh-snow density.

The authors ignore snowfall undercatch due to wind, which can be very important in windswept environments. Sublimation from the snowpack may have been overestimated, which could have contributed to the overall underestimation of snow accumulation upwind and downwind of drift accumulation areas. The density of fresh snow was estimated at 50 kg/m^3 since there was no precipitation measured on either day that was colder than $-15°C$. This may be an underestimation of density that would yield a lower threshold wind speed from Kind's (1981) threshold shear velocity equation, which would overestimate the amount of snowdrift. These issues will be presented in this discussion.

The authors use 81 days of daily data that are the same as those recorded at the Pullman 2NW National Weather Service (NWS) COOP station (ID 456789) (archived at NCDC 2004). As humidity data were not available at this site for the same period, a period of similar meteorological conditions (temperature, precipitation, and wind speed) from December 5, 2000, through February 23, 2001, from the nearby Pullman/Moscow airport (Fig. 1) was used to compute the snowpack sublimation, hereafter called the comparison study period.

The average daily wind speed for the 81 days of the study period is 3.8 m/s with 9 days of no wind. Assuming that the precipitation is measured by a precipitation gauge shielded with an Alter-shield, the total precipitation over the study period would increase from 142.5 to 161.3 mm, or 12.9% by considering wind undercatch, as per Goodison et al. (1998), and assuming that daily trace events contributed half of the measurable amount of precipitation, i.e., 0.127 mm. For the 81 days in 2000–2001, the increase would be from 91.3 to 136.7 mm, or 49.7%.

Sublimation from the snowpack can be estimated using the latent heat-flux equation, i.e., using wind speed and vapor-pressure deficit with the assumption that surface vapor pressure is saturated and can be computed from the ambient air temperature (Fassnacht 2004). A surface roughness height of 0.01 m was used, as per Walter et al. (2004). From the comparison study, the snowpack sublimation was computed to be 26.5 mm (0.327 mm/day) and occurred 37.5% of the time. Since the humidity is high in the area (the average relative humidity is 91.2%), the sublimation is likely vapor-pressure limited (the average vapor-pressure deficit was 0.468 mb). Even if the process was not vapor-pressure limited, then the latent heat flux would be an overestimate.

While Walter et al. (2004) do not present the sublimation rates, the computed snow fluxes were only a portion of the snowpack sublimation computed from the latent heat flux.

The formulation used to estimate fresh snow density could not be found in Goodison et al. (1981). Other formulations exist in the literature, as summarized by Fassnacht and Soulis (2002), all of which use higher densities than the formulation presented by the authors. While The authors' formulation decreases at temperatures colder than −15°C, Fassnacht and Soulis (2002) suggested that the density of fresh snow may actually increase at temperatures colder than −16°C due to a general decrease in snow crystal size at cold formation temperatures. Using a fresh snow density, such as presented by Hedstrom and Pomeroy (1998), resulted in only 23.4% of the snowdrifting flux computed by Walter et al. (2004).

Snow data are available from the Pullman, Wash., site. Using these data and data from surrounding NWS stations, the observed SWE was divided by the snow depth to compute an approximate fresh snow density (Fig. 2). The plot illustrates that there is a large variation in fresh snow density, with few daily observations of 50 kg/m^3 fresh snow density.

Based on the average daily air temperature, precipitation can fall as snow at air temperatures warmer than 0°C (Fassnacht and Soulis 2002). This would increase snow accumulation.

The authors corroborate the model's snowmelt performance by comparison to observed snowmelt at four sites (Danville, Vt., Bloomville, N.Y., Easton, Minn., Troy, Ind.). However, the snowdrift model is not used in these areas, and while it is used for Pullman, the snowmelt component of the model is not used. A simulation of the model's snowpack performance for snowdrifting plus accumulation and snowmelt at the same site would better illustrate the model's capabilities and the integration of the simple snowdrift formulation within the distributed hydrological model, as per the paper's title and its objectives.

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Closure to "Simple Snowdrift Model for Distributed Hydrological Modeling" by M. Todd Walter, Donald K. McCool, Larry G. King, Myron Molnau, and Gaylon S. Campbell

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We appreciate the discussion of Fassnacht and Brazenec, which emphasizes the difficulty in parsimonious modeling on snow processes and suggests some potential improvements to the approaches we used in our snowdrift study. They speculate that our study may have underestimated snowfall, overestimated snowdrift, and overestimated sublimation. While these may be true, our methods were not without justification, as discussed below.

With regard to snowfall, we are of course aware of the commonly employed correction factors but chose not to use them here because ground measurements of new snow generally agreed well with gauge measurements. Admittedly, we made few such comparisons but the limited data we had did not suggest the need for a correction. Also, although Fassnacht and Brazenec note that snow may fall at temperatures greater than 0°C, it is also true that rain may fall at temperatures less than 0°C, so we did not think that 0°C was an unreasonable temperature at which to partition precipitation into rain and snow.

Fassnacht and Brazenec suggest that we underestimated the density of new-fallen snow, which would lead to an overestimation of snowdrift. They also correctly note that the equation we used to estimate new snow density is not in Goodison et al. (1981); indeed, we modified an equation by Barry et al. (1990) and "mined" data from Goodison et al. (1981) and other parts of the *Handbook of Snow* (Gray and Male 1981) to test our relationship. Barry et al. (1990) proposed

$$\rho_{ns} = 50 + 1.7(T_w + 15)^{1.5} \tag{1}$$

where ρ_{ns}=new-snow density (kg m^{-3}) and T_w=wet-bulb temperature. We found that, for the range of temperatures we observed, $(T_w+15)^{1.5}$ was linearly correlated with $(T+15)$ with a regression slope of two; T is air temperature. Thus we proposed

$$\rho_{ns} = 50 + 3.4(T + 15) \tag{2}$$

Note, we used T_w at saturated air-vapor density in our regression, and we recognize that Eq. (2) is a rough approximation. However, there is tremendous scatter in the ρ_{ns} data (e.g., Fassnacht and Brazenec Fig. 2), and it is unlikely that any equation as simple as Eqs. (1) or (2) will meaningfully capture the observed variability—in fact, Eq. (2) lies well within the "data cloud." Goodison et al. (1981) also show that the new-snow density changes within hours of falling, so it was not obvious how to estimate this characteristic with meaningful precision. We agree with Fassnacht and Brazenec that, when data are available, it is best to use snow measures directly rather than approximations like Eqs. (1) and (2).

We admittedly handled sublimation in a crude and unsatisfactory way. We assumed large sublimation of the drifting snow. Specifically, and based loosely on Tabler (1975) and Pomerory and Li (2000), we assumed sublimation was proportional to the wind's carrying capacity as long as there was enough new snow available. We are not sure how Fassnacht and Brazenec concluded that Pullman, Wash., was a humid area (average relative humidity =91%), but the Western Regional Climate Center Web site shows monthly average regional humidity rarely exceeds 80%. Regardless, their point is well taken that sublimation should be modeled more mechanistically than we did in our paper.

Fassnacht and Brazenec also noted that we corroborated our snowmelt model with data from locations other than Pullman. Note that "Troy, Ind.," is a mistake in the journal article; the location should be Troy, Id., which was ~30 km from the site where drift measurements were made. We direct readers to Walter et al. (2005) for complete evaluation of the melt model. Incidentally, while on the topic of errata, Eq. (7) should be

$$\Delta u_{\text{lee}} = -\exp\left(\frac{4(Z_{\max} - Z_i)}{Z_{\max}}\right)$$

Our mistake.

Final Thoughts

We are heartened by the increasing frequency with which mechanistic hydrological models are being developed, used, and improved in lieu of temptingly simpler black-box models. We would like to thank Fassnacht and Brazenec for their thoughtful comments and we acknowledge Fassnacht's and colleagues' substantial contributions to the field of snow hydrology. With respect to snowdrifting, we also recommend the excellent publications of Marks, Winstral, and their coauthors.

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