



## Improving runoff risk estimates: Formulating runoff as a bivariate process using the SCS curve number method

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[1] The Soil Conservation Service curve number (SCS-CN) method is widely used to predict storm runoff for hydraulic design purposes, such as sizing culverts and detention basins. As traditionally used, the probability of calculated runoff is equated to the probability of the causative rainfall event, an assumption that fails to account for the influence of variations in soil moisture on runoff generation. We propose a modification to the SCS-CN method that explicitly incorporates rainfall return periods and the frequency of different soil moisture states to quantify storm runoff risks. Soil moisture status is assumed to be correlated to stream base flow. Fundamentally, this approach treats runoff as the outcome of a bivariate process instead of dictating a 1:1 relationship between causative rainfall and resulting runoff volumes. Using data from the Fall Creek watershed in western New York and the headwaters of the French Broad River in the mountains of North Carolina, we show that our modified SCS-CN method improves frequency discharge predictions in medium-sized watersheds in the eastern United States in comparison to the traditional application of the method.

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### 1. Introduction

[2] The Soil Conservation Service (currently the Natural Resources Conservation Service) curve number (SCS-CN) [U.S. Soil Conservation Service (SCS), 1972] method is widely used to predict runoff quantities in ungauged basins. While more sophisticated methods are available, its simplicity and dependence on readily available catchment properties has contributed to its continued popularity, particularly among practicing water resource engineers [Ponce and Hawkins, 1996; Garen and Moore, 2005]. The basic form of the SCS-CN rainfall-runoff relationship appears to be logical in that no runoff occurs until a threshold in rainfall is met and then, the fraction of rainfall contributing to runoff increases as the rainfall amount becomes larger:

$$Q = \frac{P^2}{P - S} \quad (1)$$

where  $Q$  is event discharge volume (mm),  $P$  is effective rainfall (mm), and  $S$  is a conceptual available soil storage volume (mm) calculated from

$$S = \frac{25,400}{Cn} - 254 \quad (2)$$

where  $Cn$  varies between 0 (no runoff generation) and 100 (all rain produces runoff) and is traditionally based on

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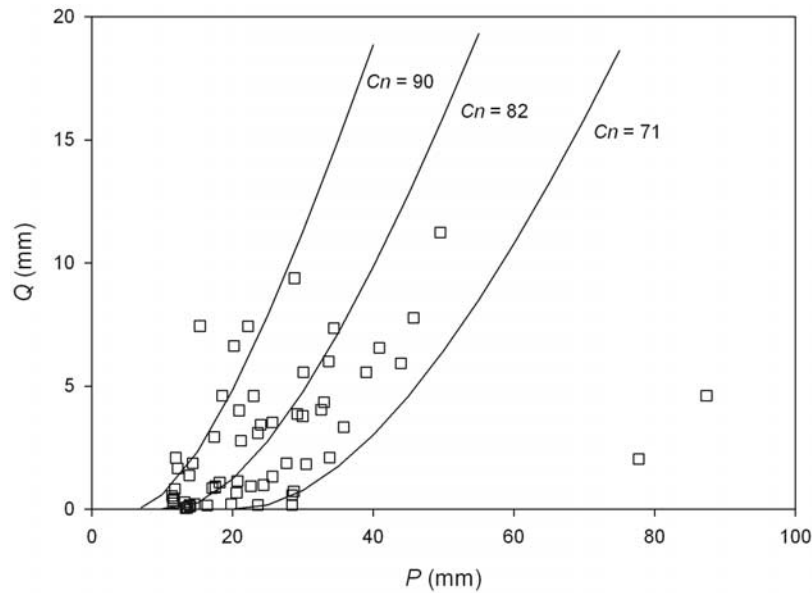
catchment land use and soil type via published tables [e.g., SCS, 1972]. Most hydrologic engineering texts and manuals still suggest that the  $Cn$  can be adjusted to account for wetter or drier conditions as determined by the 5-day antecedent rainfall although the Natural Resources Conservation Service has discounted the accuracy of using antecedent rainfall to adjust the  $Cn$ .

[3] For this standard formulation of the SCS-CN method, the probability of a  $Q$  of a given magnitude,  $Q_i$ , is related to the probability of the causative  $P$  of a given magnitude  $P_i$ :

$$\Pr(Q = Q_i) = \Pr(P = P_i) \text{ where } Q_i = g(P_i) \quad (3)$$

where  $g$  indicates a function relating  $Q$  and  $P$ , in this case  $g$  = equation (1).

[4] However, a limitation of this standard formulation stems from the hydrological reality that  $S$  itself should not be an immutable parameter but one that should vary with changes in catchment soil moisture storage, a fact only grossly captured by the antecedent rainfall adjustment. Recent work has focused on refining the SCS-CN method for more conceptually coherent use in continuous watershed models by incorporating an underlying soil moisture accounting scheme [i.e., Michel *et al.*, 2005]. But, in many engineering design applications of the SCS-CN method, say, sizing a culvert, engineers are interested in estimating a given discharge associated with a desired return period. If the variability of the  $S$  due to antecedent wetness conditions is ignored when estimating event-based probabilities of runoff, the SCS-CN method implies that the probability of a given runoff quantity is dictated only by the probability of the causative precipitation input, as indicated in (3), an



**Figure 1.** Precipitation-discharge plot for 60 events during 2000–2005 (symbols) in the Fall Creek watershed near Ithaca, New York. The three solid lines indicate the SCS-CN equation (equation (1)) with  $Cn$  values of 90, 82, and 71 from left to right; note that  $P = P_{Tot} - 0.2S$ .

assumption countered by studies of flood processes [Merz *et al.*, 2006]. Additionally, precipitation-discharge ( $P$ - $Q$ ) plots many times reveal no consistent 1:1 relationship between  $P$  and  $Q$ , e.g., a single precipitation amount can have multiple resulting discharges. Sizable scatter in  $P$ - $Q$  pairs is frequently noted in the literature, many times leading to rank ordering and recombining of  $P$  and  $Q$  observations so each value in the pair has the same return period instead of preserving the natural  $P$ - $Q$  pairing [e.g., Hawkins, 1993]. As an example of a  $P$ - $Q$  plot with scatter, Figure 1 shows data from the Fall Creek watershed near Ithaca, New York (additional details on the selection of these  $P$ - $Q$  pairs is given below). To give a sense of the variation in  $Cn$  that capture these data, we best fit (1) to the data ( $Cn = 82$ ) and calculated  $Cn$  that envelope 80% of the points, corresponding to  $Cn = 71$  (bottom line in Figure 1) and  $Cn = 90$  (top line in Figure 1).

[5] Thus, while hydrologic engineers have generally recognized that the antecedent moisture state influences storm runoff quantity, nobody has satisfactorily proposed a means to include the moisture state in the determination of runoff risk. McCuen [2002] suggested treating the  $Cn$  (or  $S$ ) as a random variable, identifying a suitable distribution and deriving confidence intervals. De Michele and Salvadori [2002] incorporated soil moisture into the calculation of the frequency distribution of peak floods but only calculated the probability of being in one of the three published antecedent moisture conditions (AMC) as based on antecedent rainfall. Conversely, Mishra and Singh [2006] developed a relationship between  $S$  and 5-day antecedent rainfall for watersheds in India but did not try to relate variations in  $S$  to changes in risk of a given  $Q$ . Building on elements of this previous work, we contend that  $S$  should vary with a watershed's antecedent wetness [e.g., McCuen, 2002], that the distribution of  $S$  can be deterministically developed [e.g., Mishra and Singh, 2006], and that  $S$  can be incorporated into

estimates of risk of runoff, as done in a limited way by De Michele and Salvadori [2002]. However, differing from this previous work, we suggest that  $S$  is best linked to base flow immediately preceding the causative precipitation event,  $Q_{base}$ , instead of the 5-day antecedent rainfall, at least in temperate regions with large intra-annual variations in evapotranspiration. For a catchment in Belgium, Troch *et al.* [1993] related streamflow to catchment water table height using Boussinesq groundwater theory, thereby indicating the possibility of using base flow as a proxy for  $S$  within the framework of the SCS-CN method.

[6] The central aim of this paper is to present a simple approach for modifying the SCS-CN method to make more robust estimates of runoff risk. This modified method treats runoff generation ( $Q$ ) as the outcome of a bivariate process involving interaction between watershed storage,  $S$ , which is a function of antecedent moisture conditions in the watershed, and precipitation,  $P$ . Treating the magnitude of  $Q$  as the outcome of a bivariate process:

$$\Pr(Q = Q_i) = \int \int_{M(Q_i)} f_{P,S}(P_i, S_i) dP_i dS_i$$

where

$$M(Q_i) \in \{(P_i, S_i) | h(P_i, S_i) = Q_i\} \quad (4)$$

and  $f_{P,S}$  is a bivariate probability density function and  $h$  is a function relating both  $P$  and  $S$  to  $Q$ , (1) in this case. Fundamentally, (4) calculates the volume (probability) under regions of the joint probability distribution where the given pairing of  $P$  and  $S$  result in a desired  $Q$ .

[7] Quantification of hydrologic events using bivariate formulations has started to gain favor in the last decade but has largely been limited to relating flood peak discharge and

event volume [see *Yue and Rasmussen*, 2002]. Here, we handle the interaction between more fundamental watershed processes using such a bivariate formulation.

## 2. New SCS-CN Method Explained

[8] If the occurrence of  $P$  and  $S$  are independent (as we assume here since we look at them over short time spans) and the variables treated as discrete quantities, (4) can be simplified to

$$\Pr(Q = Q_i) = \sum_{M(Q_i)} \Pr(S = S_i) \times \Pr(P = P_i)$$

where

$$M(Q_i) \in \{(P_i, S_i) | h(P_i, S_i) = Q_i\} \quad (5)$$

As applied here, the function  $h$  is given by (1). Equation (5) is sometimes alternately composed in terms of probability of nonexceedance as the so-called total probability theorem. This approach is used by *Dyhouse* [1985] in calculating coincident flooding at the junction of rivers and by *Pingel and Ford* [2004] in estimating internal flooding in areas surrounded by levees.

[9] Application of (5) requires knowledge of the frequency distribution of daily values of  $S$  and  $P$ . The frequency distribution of rainfall amounts can be readily obtained from a frequency analysis of historical rainfall records; the results of such analyses are often published (e.g., G. M. Bonnin et al., *Precipitation-frequency atlas of the United States*, in NOAA Atlas 14, vol. 2, version 3, unpublished manuscript, 2004, available at <http://www.nws.noaa.gov/oh/hdsc/currentpf.htm>). However, how to determine the frequency distribution of  $S$  is not well established. Following the success of *Troch et al.* [1993] in relating streamflow to catchment water table height, we propose developing a relationship between  $S$  values, back calculated from observed  $P$ - $Q$  pairs, and corresponding base flow,  $Q_{base}$ , immediately before the event. The resulting  $S(Q_{base})$  function can be used to transform stream base flow discharge records to a series of  $S$  values from which the frequency distribution of  $S$  can be determined, i.e., the percentage of time at which different  $S$  values occur. The methodology explained here is intentionally general, but the subsequent example applications provide some specific methodological details that may be directly transferable to other watersheds.

## 3. Example Applications

### 3.1. Data and Site Descriptions

[10] We use two basins of differing climate and watershed characteristics to illustrate the treatment of runoff generation as a bivariate process. The Fall Creek watershed is a 328 km<sup>2</sup> rural basin in hilly terrain located in western New York. The upper French Broad River watershed is a 174 km<sup>2</sup> forested basin located in the mountains of North Carolina. Average annual rainfall is ~100 cm in the Fall Creek watershed and >200 cm in the upper French Broad River. For both basins, we use daily "events" reported on a fixed 24-h interval because 24-h precipitation data are most commonly used in engineering hydrologic design. Also, most event runoff is

routed through the two watersheds within 24 h, so this use of a single day of daily average discharge seemed a suitable estimate for total event runoff volume. We calculated the event discharge corresponding to annual maximum discharge for a range of return periods.

[11] For Fall Creek, discharge data were from the USGS gauge near Ithaca, New York (04234000) and rainfall data were based on the average of three National Weather Service gauges (Freeville 1 NE-NYS Coop 303050, Locke 2 W-NYS Coop 304836, Cornell University-NYS Coop 304174) available from the Northeast Regional Climate Center CLIMOD database. For the French Broad River, discharge data were from the USGS gauge near Rosman, North Carolina (03439000) and rainfall data were based on the average of two National Weather Service gauges (Rosman-Coop 314786, Lake Toxaway 2 SW-Coop 314788) available from the United States National Climatic Data Center.

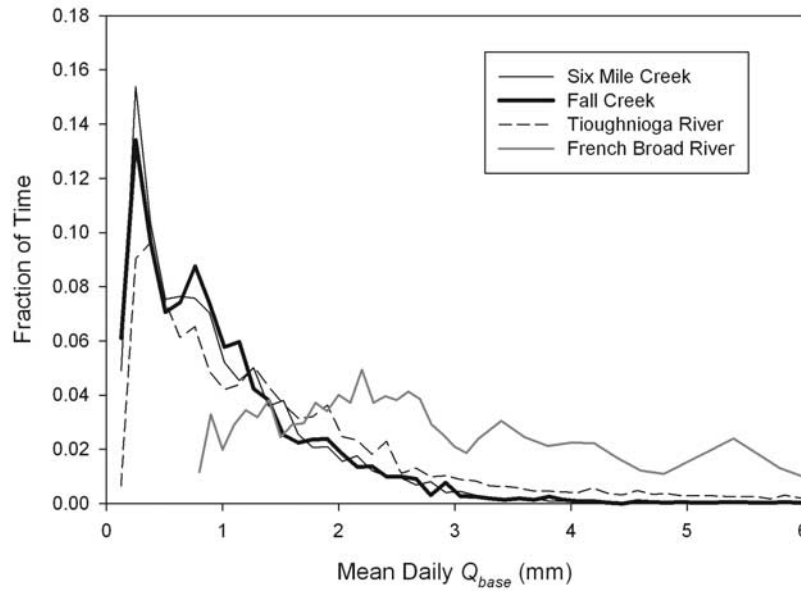
[12] We primarily selected storm events that were distinct in time with no precipitation for at least 2 days before and 2 days after. Additionally, we only considered storm events for which precipitation readings were within 25% of their average. When precipitation readings diverged more greatly, we did not include the event, assuming we did not have an accurate measure of heterogeneities in rainfall given the limited available rain gauge network. Other researchers have noted errors in rainfall-runoff model output due to uncertainty in rainfall inputs [e.g., *Andreassian et al.*, 2001; *Huard and Mailhot*, 2006]. Note, while we effectively look only at spatially uniform rainfall events, this is not our motivation for using like rain gauge readings. Instead, our interest is to limit variability in  $Q$  (for the same causative  $P$ ) to the soil moisture status, not uncertainty in  $P$ . If a more robust rain gauge network was available, we would have weighted the spatially heterogeneous rainfall.

### 3.2. Specific Procedure

[13] We used the following steps to develop risk estimates of runoff for the two basins:

[14] 1. Develop the rainfall frequency histogram (in the sense that the raw number of each rainfall occurrence is normalized by total number of occurrences): We used published rainfall frequency relationships from Bonnin et al. (unpublished manuscript, 2004) to construct a histogram of the annual probability of a given 24-h rainfall amount. This entailed converting probability of exceedances associated with return periods to an annual probability of occurrence of a specific rainfall quantity. For example, a 10-year return period event has a 0.10 probability of being equaled or exceeded in a given year, but a storm around the size of a 10-year event only has a 0.06 probability of occurring in a given year since in some years a larger storm may occur. Thus, the 100-year event annually occurs 1% of the time, the 50-year event annually occurs 1% of the time, the 25-year event annually occurs 2% of the time, the 10-year event annually occurs 6% of the time, etc. Notably, at small return periods, a single rainfall value will span a large percentile range. We did not consider this critical here since we are interested in the largest flood events, but one could develop a more refined histogram if needed.

[15] 2. Establish the base flow ( $Q_{base}$ ) frequency histogram (Figure 2): We used the 1995 to 2005 discharge record and extracted base flow using the local minimum method as

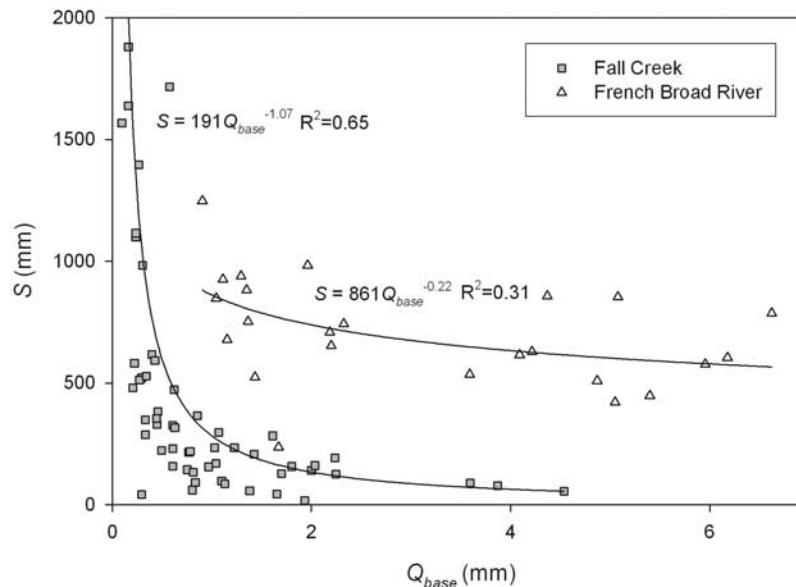


**Figure 2.** Base flow for Fall Creek, New York (thick solid line), and the French Broad River, North Carolina (thin gray solid line). Also shown is base flow for streams near Fall Creek: Tioughnioga River, New York (dashed line), and Six Mile Creek, New York (thin black line). Base flows are determined from 1995 to 2005 daily discharge records.

automated in the Web-based hydrograph analysis tool [Lim *et al.*, 2005].

[16] 3. Establish the  $S$  versus  $Q_{base}$  relationship (Figure 3): For Fall Creek, we used 60 event pairs of daily  $P$  and  $Q$  from water years 2000 to 2005 (the same pairs are shown in Figure 1). For the French Broad River, we used 25 pairs of  $P$  and  $Q$  from water years 1995 to 1999. We back calculated  $S$  by rearranging (1), assuming  $P$  is equal to the total event precipitation ( $P_{Tot}$ ). This effectively establishes an initial abstraction (the minimum amount of rainfall necessary before runoff starts) of zero. We considered this a reasonable simplification since we focus on precipitation from

larger events ( $>10$  mm) where any potential initial abstraction would only be a small fraction of  $P_{Tot}$ . Additionally, in these larger sized basins, some rainfall will fall directly on surface water or riparian areas with an immediate (albeit small) response in the mainstream hydrograph. We use storms events with  $P_{Tot} > 10$  mm to minimize uncertainty in the determination of the storm event discharge. Typically, anywhere from only 0.1% to 10% of rainfall is converted to event runoff; thus, small rainfall events result in even smaller changes in runoff that can sometimes be difficult to discern in the discharge time series.



**Figure 3.**  $S$  versus  $Q_{base}$  for the Fall Creek, New York, and French Broad River, North Carolina, watersheds.

**Table 1.** Fall Creek Watershed Example Calculation Table<sup>a</sup>

<i>Cn</i>	<i>S<sub>i</sub></i>	<i>Q<sub>i</sub></i> (mm)						
		<i>P<sub>i</sub></i> = 52 mm	<i>P<sub>i</sub></i> = 57 mm	<i>P<sub>i</sub></i> = 73 mm	<i>P<sub>i</sub></i> = 85 mm	<i>P<sub>i</sub></i> = 101 mm	<i>P<sub>i</sub></i> = 113 mm	<i>P<sub>i</sub></i> = 125 mm
12.7	1748	1.5	1.8	<b>3.0</b>	3.9	5.5	6.8	8.3
23.4	830	<b>3.0</b>	3.6	6.0	7.9	10.9	13.4	16.3
32.1	537	4.5	5.4	8.8	11.6	15.9	19.5	23.5
39.2	394	6.0	7.1	11.5	15.1	20.4	25.0	30.0
45.0	310	7.3	8.7	14.0	18.3	24.6	29.9	35.7

<i>Cn</i>	Fraction of Time	Pr( <i>Q</i> = <i>Q<sub>i</sub></i> )						
		Pr( <i>P</i> = <i>P<sub>i</sub></i> ) = 0.50	Pr( <i>P</i> = <i>P<sub>i</sub></i> ) = 0.30	Pr( <i>P</i> = <i>P<sub>i</sub></i> ) = 0.10	Pr( <i>P</i> = <i>P<sub>i</sub></i> ) = 0.06	Pr( <i>P</i> = <i>P<sub>i</sub></i> ) = 0.02	Pr( <i>P</i> = <i>P<sub>i</sub></i> ) = 0.01	Pr( <i>P</i> = <i>P<sub>i</sub></i> ) = 0.01
12.7	0.061	0.0305	0.0183	<b>0.0061</b>	0.0037	0.0012	0.0006	0.0006
23.4	0.134	<b>0.0672</b>	0.0403	0.0134	0.0081	0.0027	0.0013	0.0013
32.1	0.097	0.0484	0.0290	0.0097	0.0058	0.0019	0.0010	0.0010
39.2	0.071	0.0353	0.0212	0.0071	0.0042	0.0014	0.0007	0.0007
45.0	0.074	0.0371	0.0223	0.0074	0.0045	0.0015	0.0007	0.0007

<sup>a</sup>The top half of the table calculates the *Q* associated with 7 possible rainfall amounts and 5 possible *S<sub>i</sub>* (or *Cn*) values (the smallest 5 out of a total of 36 we use in the analysis). *Q* is calculated using equation (1) with the corresponding *P<sub>i</sub>* and *S<sub>i</sub>*. The bottom half of the table calculates Pr(*Q* = *Q<sub>i</sub>*) corresponding to each *Q<sub>i</sub>* calculated in the top half. Corresponding *Q<sub>i</sub>* and Pr(*Q* = *Q<sub>i</sub>*) have the same relative position in the top and bottom halves of the table. Pr(*Q* = *Q<sub>i</sub>*) is calculated using equation (5); for instance, the probability of 3.0 mm *Q* is 0.10 × 0.061 + 0.50 × 0.134 = 0.073, corresponding to the bold values in the table. The columns labeled *P<sub>i</sub>* = 52 through *P<sub>i</sub>* = 125 correspond to return periods of 1, 2, 5, 10, 25, 50, and 100 years.

[17] *Q<sub>base</sub>* was taken as the base flow on the day prior to a rainfall event and was subtracted from the event discharge to estimate *Q*. As shown in Figure 3, we plotted *Q<sub>base</sub>* against the associated *S* and fit a power law relationship to the data:

$$S = aQ_{base}^b \tag{6}$$

where *a* and *b* are fitting parameters.

[18] 4. Create frequency histogram of *S*: We use the power law relationships between *S* and *Q<sub>base</sub>* (e.g., a fitted equation (6)) to translate the 36 *Q<sub>base</sub>* values in our base flow histogram (developed in step ii, see Figure 2) to corresponding *S* values. The “fraction of time” value for any given *Q<sub>base</sub>* value was assigned to the corresponding *S* value to develop an *S* frequency histogram.

[19] 5. Calculate *Q* for each *P* – *S* combination using (1): With 7 *P* values and 36 *S* values, we end up with 252 *Q* values. We concurrently calculate the corresponding probability of the 252 runoff depths, using (5). Sample calculations for the five lowest observed *Cn* values for the Fall Creek watershed example are shown in Table 1.

[20] If the probability of nonexceedance is desired, one can rank the resulting *Q* values from lowest to highest and cumulatively sum the associated probabilities calculated from (5). The return period is obviously computed by taking the inverse of the probability of nonexceedance subtracted from one.

### 3.3. Evaluation of Predicted Runoff Probabilities

#### 3.3.1. Comparison With Observed Runoff Probabilities

[21] Perhaps the best test of our proposed bivariate SCS-CN method is to see how well it matches the probabilities of observed storm runoff. To do this, we compare estimates of return periods based on our bivariate method to return periods of a generalized extreme value (GEV) distribution and log Pearson III (LP3) distribution [Stedinger et al., 1993] fit to the observed maximum annual average daily flow for Fall Creek (Figure 4) and the French Broad River (Figure 5) with base flow subtracted. The LP3 was fit using

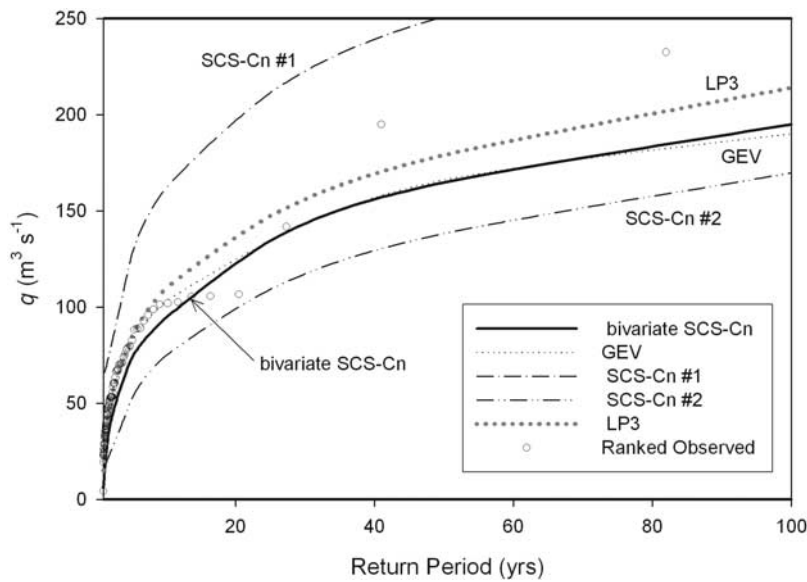
regional skew parameters [U.S. Interagency Advisory Committee on Water Data, 1982], not the gauge specific skew. The GEV and LP3 were selected on the basis of a graphical assessment of their probability plots. Probability plots are constructed by plotting the ordered observed data against the estimated discharge values for the corresponding percentile calculated using the inverse cumulative density function of the respective flood distribution function. Percentiles are calculated using Cunnane’s plotting position in the case of the GEV and Blom’s formula for the LP3 [Stedinger et al., 1993]. For both watersheds, we found that points fell along a straight line passing through the origin, a qualitative indication that the fitted flood frequency distribution is reasonable [Stedinger et al., 1993]. In Figures 4 and 5, we also include the observed data ordered using plotting positions as a simple check on the reasonability of the chosen frequency distributions.

[22] For Fall Creek, the bivariate SCS-CN method matches the GEV distribution remarkably well (Figure 4) while it underpredicts large return period values of the LP3 distribution by ~10%. For the French Broad River, the bivariate SCS-CN method slightly underpredicts the GEV and LP3 distributions, especially for low-return periods (Figure 5). Note, in Figures 4 and 5 we converted our SCS-CN runoff to  $q = Q \times \text{watershed area} \times (\text{s d}^{-1})^{-1}$ .

#### 3.3.2. Comparison With Traditional Predictions of Runoff Probabilities

[23] We also compare our bivariate-based predictions to *q* predictions based on the traditional SCS-CN approach in which *S* is estimated in two different ways. Note, in the traditional SCS-CN approach we use  $P = P_{Tot} - 0.2S$  [SCS, 1972]. First, we used a best fit *S* by fitting (1) to the observed *P*-*Q* pairs. For Fall Creek, the best fit *Cn* = 82. For the French Broad River, the best fit *Cn* = 62. For both Fall Creek and the French Broad River, this parameterization of the standard SCS-CN method systematically over predicts for both the LP3 and GEV (SCS-CN 1 in Figures 4 and 5).

[24] Alternatively, we determine *Cn* from the traditionally used tables [SCS, 1972]. For Fall Creek, we assume land use of 41% agricultural (predominantly hay, *Cn* = 68), 52%

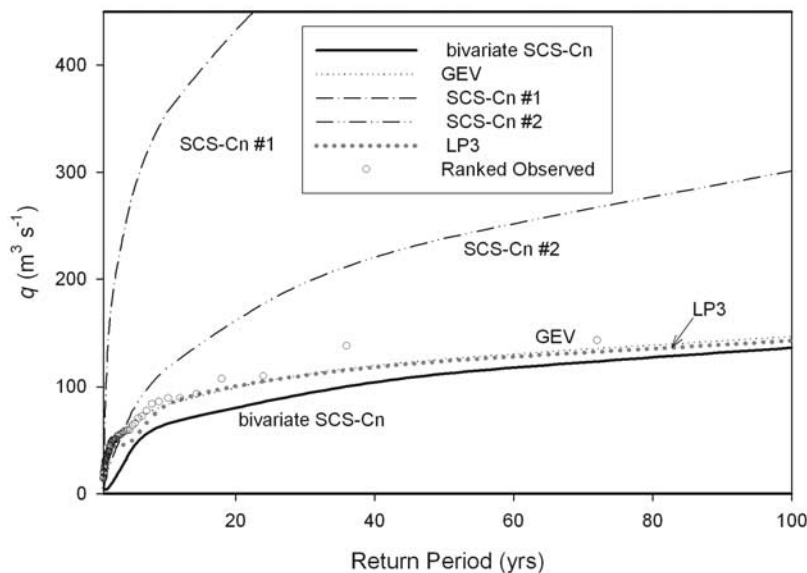


**Figure 4.** Comparison of bivariate formulation of SCS-CN (bivariate SCS-CN, equation (5), thick black line) to observed daily maximum average stream discharge (with base flow subtracted) fit to a GEV and log Pearson III distribution for the Fall Creek watershed, New York. Also included are two different traditional means of parameterizing the standard SCS-CN formulation (equation (1) and (2)): SCS-CN 1 uses a best fit  $Cn = 82$  from Figure 1, and SCS-CN 2 uses the traditional table-based  $Cn$  of 67.

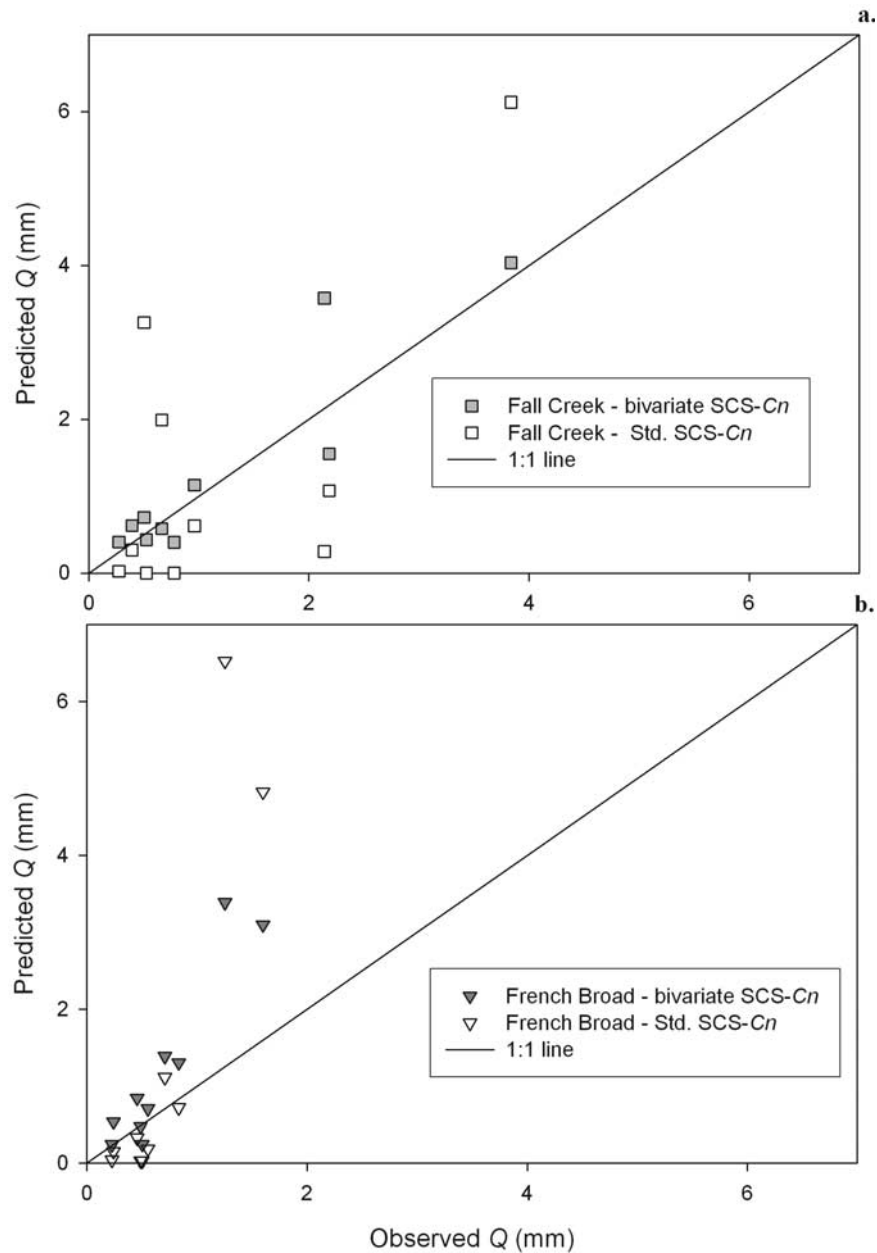
forest ( $Cn = 63$ ), and 7% urban ( $Cn = 87$ ) with a mix of Hydrologic Class B and C soils [Johnson *et al.*, 2007]; we compute a watershed-wide  $Cn$  of 67. For Fall Creek, this parameterization of the standard SCS-CN method systematically under predicts  $q$  compared to the GEV and LP3 (SCS-CN 2 in Figure 4). The upper French Broad River is nearly entirely forested with a mix of hydrologic class A and B soils [King *et al.*, 1974]. On the basis of tabulated values, we use  $Cn = 43$ . For the French Broad River, this

parameterization of the standard SCS-CN method again overpredicts  $q$  compared to the GEV and LP3 (SCS-CN 2 in Figure 5). The results presented in Figures 4 and 5 suggest that treating runoff as a bivariate process is a robust alternative to other means of parameterizing the standard SCS-CN method.

[25] Additionally, we further test our modified SCS-CN method by considering ten events for each basin not previously used to establish the power law relationships



**Figure 5.** Comparison of bivariate formulation of SCS-CN (bivariate SCS-CN, equation (5), thick black line) to observed daily maximum average stream discharge (with base flow subtracted) fit to a GEV and log Pearson III distribution for the French Broad River watershed, North Carolina. Also included are two different traditional means of parameterizing the standard SCS-CN formulation (equations (1) and (2)): SCS-CN 1 uses a best fit  $Cn = 62$ , and SCS-CN 2 uses the traditional table-based  $Cn$  of 43.



**Figure 6.** Comparison of bivariate SCS-CN (equations (5) and (6), solid symbols) and the standard SCS-CN method (open symbols) for Fall Creek, New York (Figure 6a), and the French Broad River, North Carolina (Figure 6b). For the standard SCS-CN method,  $C_n = 82$  for Fall Creek, and  $C_n = 62$  for the French Broad River. In both cases,  $C_n$  is adjusted by 0.75 to account for events when 5-day antecedent rainfall is less than 36 mm. For the standard SCS-CN, we use  $P = P_{Tot} - 0.2S$ . The line indicates a 1:1 relationship between predicted and observed values.

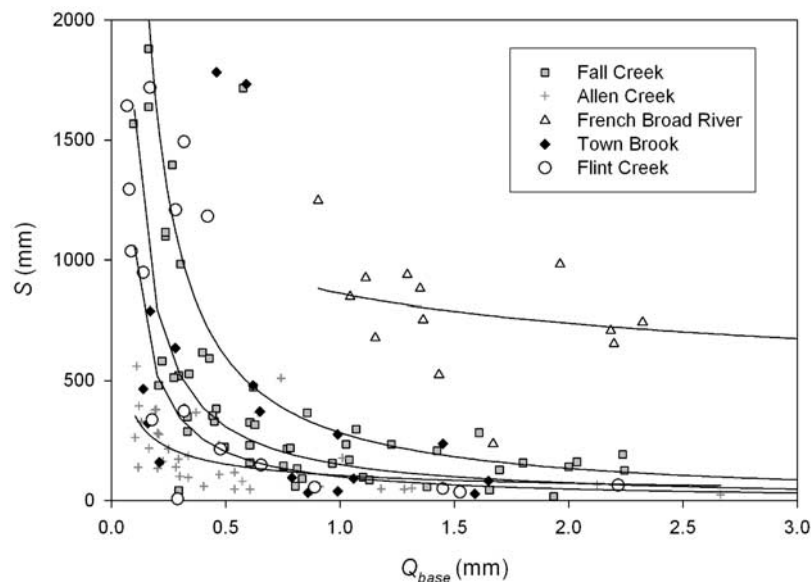
between  $S$  and  $Q_{base}$ . Using the observed  $P$  and  $Q_{base}$  as an input, the modified method favorably matches the observed discharge in most cases (Figures 6a and 6b). As a matter of comparison, we also show the standard form of the SCS-CN with a best fit  $C_n = 82$  for Fall Creek and  $C_n = 62$  for the French Broad River. If the 5-day antecedent rainfall before a storm event was  $<36$  mm, as per the *National Engineering Handbook* [SCS, 1972], the  $C_n$  was reduced by 25%, (without this adjustment, the method proves even less favorable). For Fall Creek, the root-mean-square errors (RMSE) were 0.50 (0.76) mm and 1.36 (1.56) mm for the bivariate and standard SCS-CN methods, respectively, where standard

errors are indicated in parenthesis. For the French Broad River, the RMSEs were 0.84 (1.21) mm and 1.88 (2.99) mm for the bivariate and standard SCS-CN methods, where standard errors are indicated in parenthesis. Thus, when tabulated  $C_n$  values were used, agreement between observed and predicted  $q$  were worse.

## 4. Discussion

### 4.1. Developing Regional Relationships

[26] While we are interested in improving the predictive accuracy of the SCS-CN method, we also wish to maintain



**Figure 7.** Comparison of  $S$  versus  $Q_{base}$  curves. We show three rural watersheds in upstate New York: Fall Creek, Flint Creek, and Town Brook. Additionally, we show a suburban watershed in upstate New York and the upper French Broad River, North Carolina.

its simplicity, transparency, and ease of determining inputs a priori, so that we can use this method in ungauged as well as gauged watersheds. Linking the choice of the  $S$  to the frequency distribution of base flow appears to provide a good way to incorporate antecedent moisture conditions, but we certainly require new information to predict runoff. We contend that this information could be as readily compiled as the original development of the supposed link between  $S$  and land use.

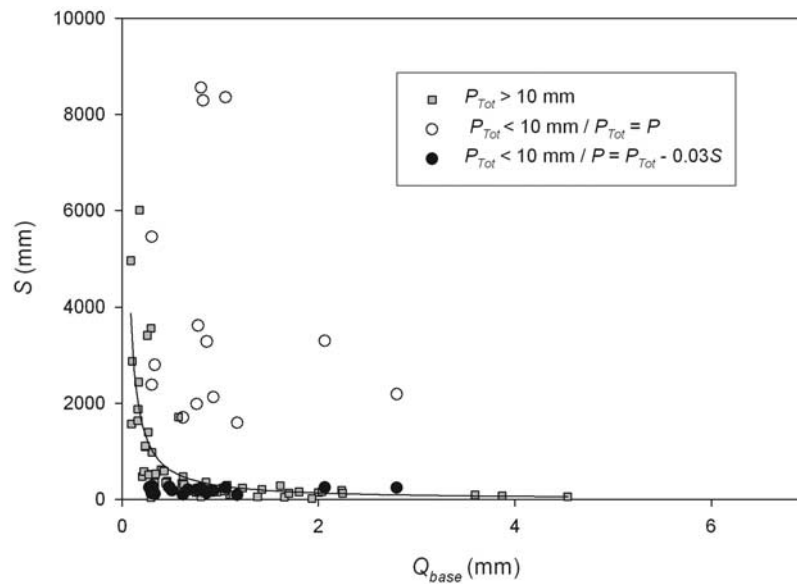
[27] For instance, base flow histograms may be regionally applicable and readily applied to ungauged basins. In Figure 2, we show base flow histograms for basins regionally near Fall Creek, namely, the 748 km<sup>2</sup> Tioughnioga River basin (USGS ID 01509000) and the 100 km<sup>2</sup> Six Mile Creek Basin, (USGS ID 04233300). The three base flow histograms are reasonably similar given the wide range in potential variation. Thus, base flow histograms from gauged watersheds could potentially be applied to nearby ungauged watersheds. Perhaps one day, direct measures of soil moisture will also provide more direct indicators of antecedent conditions. Indeed, there have been recent efforts to do this using field sensors and/or remote sensing [i.e., *Kustas et al.*, 2005].

[28] Additionally, we suggest that regional  $S$  versus  $Q_{base}$  relationships (e.g., equation (6)) could be developed for watersheds with similar compositions of dominant topographic, soil, and land use characteristics. In Figure 7, we overlay the  $S$  versus  $Q_{base}$  relationship for Fall Creek and the French Broad River with relationships for a suburban basin (70 km<sup>2</sup> Allen Creek Basin, USGS ID 04232050) and two other rural basins (36.6 km<sup>2</sup> Town Brook Basin, USGS ID 0142168; 261 km<sup>2</sup> Flint Creek Basin, USGS ID 0423520) in upstate New York. Town Brook is located approximately 150 km east of Fall Creek, near Hobart, New York, and Flint Creek is located approximately 60 km north, near Phelps, New York. As in the Fall Creek watershed, these basins consist of hilly terrain with the stream in a

broad valley bottom. The suburban Allen Creek watershed is located approximately 130 km northwest of Fall Creek on the outskirts of the city of Rochester, New York. In Figure 7, we include best fit lines for the set of points from each watershed to better illustrate the separation between different  $S$  versus  $Q_{base}$  relationships.

[29] In Figure 7, the two rural basins, despite differing in size and location in relation to Fall Creek, still have relatively similar  $S$  versus  $Q_{base}$  relationships. While many small-scale features may differ among the Fall Creek, Flint Creek, and Town Brook watersheds, the similarities in  $S$  versus  $Q_{base}$  suggest that the dominant controls on runoff generation, namely, available soil moisture storage, must be a relatively robust feature not highly sensitive to minor variations in land use, soils, or topography of the region. However, for more dramatic changes in watershed characteristics, the curves do shift. In the suburban Allen Creek basin, base flow is smaller, even when normalized for area, and  $S$  is also smaller, as expected in a watershed with extensive impervious surfaces and a system of drainage pipes directly conveying rainfall to the stream. In the French Broad River basin,  $S$  is systematically higher. This result is consistent with the deeper, well-drained soils (as implied by the hydrologic class A and B soils) in the French Broad basin relative to the New York watersheds.

[30] Given that  $S$  is presumed to represent changes in soil moisture storage, our approach for determining antecedent conditions likely works best for watersheds in which storm runoff is a saturation excess process (see *Steenhuis et al.*'s [1995] application of the SCS-CN method in this context), and it may be somewhat less effective where infiltration excess dominates. However, as suggested by examples ranging from New York to North Carolina, there is large area where this approach may apply. Additionally, the comparisons of  $S$  versus  $Q_{base}$  curves for three rural watersheds in New York indicate the relationships may not be overly sensitive to small differences in watershed character-



**Figure 8.** Illustration of influence of using an initial abstraction in establishing the  $S$  versus  $Q_{base}$  relationship for Fall Creek, New York. For points associated with  $P_{Tot} < 10$  mm with no initial abstraction (open circles),  $S$  values lie far above the dominant curve (open squares with fitted line). With a calibrated initial abstraction (0.03 $S$  in this case), points associated with  $P_{Tot} < 10$  mm (solid circles) fall onto the dominant curve established for events when  $P_{Tot} > 10$  mm (open squares).

istics and that regional curves could be developed. With the establishment of standard relationships for regions with similar dominant runoff processes and knowledge of the soil moisture frequency distribution, this bivariate SCS-CN method would permit great adaptability to localized site conditions using different combinations of rainfall probability, base flow frequency duration, and  $S$  versus  $Q_{base}$  relations.

#### 4.2. Reassessing the Initial Abstraction

[31] In the example applications of the bivariate SCS-CN method above, we assumed we could use an initial abstraction of zero ( $P = P_{Tot}$ ). To reassess this simplification, we repeated our bivariate SCS-CN method in Fall Creek with a nonzero initial abstraction. We recalculated  $S$  versus  $Q_{base}$  using 25 new  $P$ - $Q$  pairs of  $P_{Tot} < 10$  mm in addition to the 60  $P$ - $Q$  pairs of  $P_{Tot} > 10$  mm. We found that many events with  $P_{Tot} < 10$  mm deviated substantially from the rest of the  $S$  versus  $Q_{base}$  curve unless an initial abstraction was included (Figure 8). Using an initial abstraction of 0.03 $S$  (i.e.,  $P = P_{Tot} - 0.03S$ ) for all events established a suitable  $S$  versus  $Q_{base}$  relationship ( $R^2 = 0.45$ ). As a matter of comparison,  $R^2 = 0.30$  for an initial abstraction of zero and  $R^2 = 0.20$  for an initial abstraction of 0.2 $S$ , the formulation used in the standard SCS-CN method [SCS, 1972]. An initial abstraction of 0.03 $S$  is consistent with work by Jiang [2001] and much smaller than the 0.2 $S$  adjustment.

[32] Using the new  $S$  versus  $Q_{base}$  relationship that incorporated an initial abstraction of 0.03 $S$  (equation (6) with  $a = 129$  and  $b = -0.5$ ), we repeated the analysis in steps iii. to v. for the Fall Creek watershed and reestimated similar annual maximum discharges. The RMSE for annual maximum discharge estimates between the bivariate SCS-CN method without an initial abstraction and the best fitted GEV and LP3 were 7.4 (7.3)  $m^3 s^{-1}$  and 13.2 (9.0)  $m^3 s^{-1}$ ,

respectively, where standard errors are indicated in parenthesis. The RMSE for annual maximum discharge estimates between the bivariate SCS-CN method with an initial abstraction and the best fitted GEV and LP3 were 7.1 (7.4)  $m^3 s^{-1}$  and 9.8 (8.2)  $m^3 s^{-1}$ , respectively, where standard errors are indicated in parenthesis. Because of the similarity in annual maximum discharge estimates when both an initial abstract was and was not used, the original simplification of using  $P = P_{Tot}$  seems reasonable as long as we consider only  $P_{Tot} > 10$  mm. Large rainfall amounts in the French Broad River basin made it too difficult to find a sufficient number of small daily  $P$  values on which to assess the sensitivity of calculations to the initial abstraction in the French Broad River watershed.

[33] Evaluation of additional watersheds is needed to establish specific guidelines on calculating an initial abstraction. In watersheds on the order of 100  $km^2$ , neglecting an initial abstraction seems to result in suitable estimates of discharge for some applications, as illustrated for Fall Creek. However, as noted earlier, the initial abstraction quantifies the minimum amount of precipitation necessary before runoff begins. In smaller watersheds, particularly with a limited amount of perennial stream or wetland area, the initial abstraction may become more important since there will be few zones where rainfall is immediately transformed to runoff, and runoff production will be more of a threshold process. Additionally, unlike the standard SCS-CN method in which a fixed initial abstraction of 0.2 $S$  is assumed for all watersheds, using the  $S$  versus  $Q_{base}$  relationship to determine an appropriate initial abstraction may help reveal fundamental hydrologic characteristics of the basin.

#### 4.3. The 5-Day Antecedent Rainfall

[34] Last, we also consider the suitability of using  $Cn$  versus 5-day antecedent rainfall, the traditional approach in

the SCS-CN method and the one adopted by *Mishra and Singh* [2006] to develop a relationship with storage. We replicated Figure 3 for Fall Creek but used 5-day antecedent rainfall in place of  $Q_{base}$  and found no coherent relationship (not shown), only that several events with no antecedent rainfall corresponded to large  $S$  values. We suggest that in temperate regions with large swings in the degree of evapotranspiration over the course of a year, similar 5-day antecedent rainfall amounts may correspond to greatly differing soil moisture contents.

## 5. Conclusions

[35] We propose a new paradigm for using the SCS-CN method that quantifiably accounts for antecedent conditions in a watershed to determine runoff return period. Traditional uses of the SCS-CN method assumed that the runoff frequency was equal to the rainfall frequency even though rudimentary hydrologic science clearly shows this assumption to be erroneous. Our proposed method assumes a bivariate process that combines precipitation frequency and the frequency of different degrees of soil wetness. The latter we assume is related to base flow conditions prior to a storm event. We demonstrate that our proposed, bivariate-based approach to the SCS-CN method more accurately predicts runoff risk than traditional approaches, especially those that use widely adopted  $Cn$  tables.

[36] We propose that future work should adopt our proposed method to develop  $S$  (or  $Cn$ ) versus  $Q_{base}$  relationships for regional watersheds with varying distributions of runoff-controlling characteristics, e.g., soils, land use, topography, etc. These regional “master relationships” could be applicable to ungauged watersheds so that the SCS-CN method can continue to be used for its intended purpose.

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